CP3-07-23
August 2007

An Action for Chan-Paton Factors

Florian Payen

Center for Particle Physics and Phenomenology (CP3)
Institute of Nuclear Physics
Department of Physics, Université catholique de Louvain
Chemin du Cyclotron 2, B-1348 Louvain-la-Neuve, Belgium
E-mail: `florian_payen@hotmail.com`

Abstract

We show that Chan-Paton factors can be derived from a classical action describing the dynamics of a new group-valued degree of freedom attached to the boundary of an open bosonic string. We discuss the free and the interacting string in the oriented and unoriented cases, as well as the coupling of the string to an external Yang-Mills gauge field, and recover by this approach well-known results.

1 Introduction

Chan-Paton factors [1] are matrices λ_{ij} associated with the quantum states of an open bosonic string, in order to provide them with an internal non-abelian symmetry. Indeed, in a string interaction process, the factors corresponding to the external states are collected into traces which are inserted in the amplitude, leading to non-abelian gauge interactions [2, 3]. The i and j indices of the Chan-Paton factors are usually interpreted as quantum numbers attached to both ends of the string, and propagating freely during the interaction. In this paper, we want to show that this picture could be implemented in a natural way by deriving the Chan-Paton construction from the dynamics of some new degree of freedom living on the boundary of the string world-sheet.

A few attempts to put degrees of freedom on the edges of a string can be found in the literature. In particular, Marcus and Sagnotti [4] proposed successfully to quantize d free fermions on the string boundary, recovering Chan-Paton factors for the symmetry group $SO(2^{d/2})$. But in their approach the group structure only becomes apparent at the quantum level.

Here we chose a different approach, starting from a classical action introduced by Balachandran, Borchardt and Stern [5] to describe the interaction of a pointlike non-abelian charge with a Yang-Mills gauge field. The canonical quantization of this system gives rise to a finite dimensional Hilbert space carrying an irreducible unitary representation of the gauge group [5, 6], while the related path integral provides a representation for the Wilson loop of the gauge field [7, 8]. Later, Barbashov and Koshkarov [9] coupled this action to the boundary of an open bosonic string in order to describe its classical interaction with a Yang-Mills gauge field. The purpose of this paper is to show that at the quantum level this coupling leads to Chan-Paton factors.

The presentation will be organized as follows. First we will present the classical action for the new 'Chan-Paton' degree of freedom attached to the boundary of an oriented open bosonic string. Next we will show that the quantum dynamics of this degree of freedom leads to the usual Chan-Paton factors, both in the canonical quantization of a free string and in the path integral description of string interactions. Then we will extend our analysis to the case of an unoriented open bosonic string. Finally we will discuss the interaction of a string with an external Yang-Mills gauge field.

2 Action

Let us consider a Minkowski spacetime M^D with dimension D fixed to the critical value 26, described by the canonical coordinates x^μ ($\mu = 0, 1, \ldots, D-1$) and the metric

$$\eta_{\mu\nu} = diag(-1, 1, \ldots, 1). \tag{1}$$

We choose units such that $c = 1$.

An oriented open bosonic string in M^D is characterized by an oriented world-sheet Σ, parameterized by local coordinates σ^α ($\alpha = 0, 1$), which admits a boundary $\partial\Sigma$ with the induced orientation, described by some function $\bar{\sigma}^\alpha(\tau)$ of a parameter τ. The world-sheet is provided with an intrinsic metric $\gamma_{\alpha\beta}(\sigma)$ of signature $(-,+)$, and embedded by coordinates $x^\mu(\sigma)$ in M^D. The dynamics of the string is determined by the Polyakov action

$$S_\Sigma[\gamma, x] = -\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\ \sqrt{-\det\gamma}\ \gamma^{\alpha\beta}\eta_{\mu\nu}\partial_\alpha x^\mu\partial_\beta x^\nu, \tag{2}$$

where α' is the Regge slope. It is invariant under Poincaré transformations of the spacetime coordinates, orientation preserving world-sheet diffeomorphisms and Weyl transformations of the metric.

Now let G be a compact connected Lie group with dimension N_G, $\mathcal{G}$ the Lie algebra of G, and $\mathcal{H}$ a Cartan subalgebra in $\mathcal{G}$. We attach a new 'Chan-Paton' degree of freedom $g(\tau)$ in G to the boundary $\partial\Sigma$ of the string, whose dynamics is controlled by a new term added to the action [5, 9]

$$S_\Sigma[\gamma, x, g] = S_\Sigma[\gamma, x] + \int_{\partial\Sigma} d\tau\ \kappa\big(-ig^{-1}\partial_\tau g\big). \tag{3}$$

Here $\kappa(.)$ is an element of the dual $\mathcal{G}^*$ of $\mathcal{G}$. Thanks to the right action of G on $g(\tau)$, it can be univocally reduced to an element of the dual $\mathcal{H}^*$ of $\mathcal{H}$ belonging to the positive Weyl chamber defined by a family of simple roots. The boundary term preserves the symmetries of the action, and is invariant

$$S_\Sigma[\gamma, x, g^U] = S_\Sigma[\gamma, x, g] \tag{4}$$

under the left action of G on $g(\tau)$

$$g(\tau)\ \rightarrow\ g^U(\tau) = Ug(\tau). \tag{5}$$

3 The Free String

Let us first consider a free oriented open bosonic string, characterized by a topologically trivial world-sheet parametrized by (τ, σ), with boundaries described by $(\tau, 0)$ and (τ, π), and let us study the dynamics of Chan-Paton degrees of freedom $g_0(\tau)$ and $g_\pi(\tau)$ attached to its ends

$$S[\gamma, x, g_0, g_\pi] = S[\gamma, x] + \int d\tau\ \kappa\big(-i{g_0}^{-1}\partial_\tau g_0\big) - \int d\tau\ \kappa\big(-i{g_\pi}^{-1}\partial_\tau g_\pi\big). \tag{6}$$

Let $(T_a)_{a=1,\dots,N_G}$ be a basis in $\mathcal{G}$, satisfying the algebra $[T_a, T_b] = if_{ab}{}^cT_c$, where the $f_{ab}{}^c$ are structure constants. The classical equations of motion for $g_0(\tau)$ and $g_\pi(\tau)$ [5, 9]

$$\partial_\tau G_{0a}(\tau) = 0, \qquad \partial_\tau G_{\pi a}(\tau) = 0 \tag{7}$$

express the conservation of the Noether charges

$$G_{0a}(\tau) = -\kappa\big({g_0}^{-1}(\tau)T_a g_0(\tau)\big), \qquad G_{\pi a}(\tau) = \kappa\big({g_\pi}^{-1}(\tau)T_a g_\pi(\tau)\big) \tag{8}$$

associated with the left action of G on the degrees of freedom

$$g_0(\tau)\ \rightarrow\ {g_0}^U(\tau) = Ug_0(\tau), \qquad g_\pi(\tau)\ \rightarrow\ {g_\pi}^U(\tau) = Ug_\pi(\tau). \tag{9}$$

At the quantum level, choosing units such that $\hbar = 1$, the corresponding operators $\hat{G}_{0a}$ and $\hat{G}_{\pi a}$ satisfy the algebra

$$[\hat{G}_{0a}, \hat{G}_{0b}] = if_{ab}{}^c\hat{G}_{0c}, \qquad [\hat{G}_{\pi a}, \hat{G}_{\pi b}] = if_{ab}{}^c\hat{G}_{\pi c}, \tag{10}$$

and generate some representation of G on the Hilbert space of quantum states.

As stated in [5, 6], the canonical quantization of the system restricts the possible values of $\kappa(.)$. Indeed, the sector associated with $g_0(\tau)$ can be quantized if and only if $\kappa(.)$ corresponds to the highest weight of an irreducible unitary representation R of G on a vector space V with dimension N_R. Then the sector associated with $g_\pi(\tau)$ can be quantized similarly, as $-\kappa(.)$

corresponds to the lowest weight of the dual representation R^* of G on the dual space V^*, conjugated to its highest weight by some element of the Weyl group of $\mathcal{H}$. If this condition is satisfied, the Hilbert spaces of the two sectors are finite dimensional and coincide with V and V^*, while the operators $\hat{G}_{0a}$ and $\hat{G}_{\pi a}$ are identified with the generators of the representations R and R^*.

Thus the quantum states of the string coupled to Chan-Paton degrees of freedom take the form $\lambda_{ij}\,|X\rangle$, where λ_{ij} $(i,j=1,\ldots,N_R)$ is a tensor in $V\otimes V^*$ and $|X\rangle$ is a naked string state. The operators $\hat{G}_{0a}$ and $\hat{G}_{0a}$ act on these states following

$$\begin{aligned} \hat{G}_{0a}\lambda_{ij}\,|X\rangle &= R(T_a)_{ii'}\lambda_{i'j}\,|X\rangle \\ &= (R(T_a)\lambda)_{ij}\,|X\rangle, \end{aligned} \tag{11}$$

$$\begin{aligned} \hat{G}_{\pi a}\lambda_{ij}\,|X\rangle &= -R^*(T_a)_{jj'}\lambda_{ij'}\,|X\rangle \\ &= (-\lambda R(T_a))_{ij}\,|X\rangle, \end{aligned} \tag{12}$$

and generate their transformations under the left action of G

$$\begin{aligned} \lambda_{ij}\,|X\rangle \;\to\; {\lambda_{ij}}^U\,|X\rangle &= R(U)_{ii'}R^*(U)_{jj'}\lambda_{i'j'}\,|X\rangle \\ &= (R(U)\lambda R^\dagger(U))_{ij}\,|X\rangle, \end{aligned} \tag{13}$$

where the operators $R(U)$ in V form a subgroup of $U(N_R)$.

We finally recover the usual Chan-Paton factors λ_{ij} associated with the quantum states of an oriented open bosonic string [2, 3].

4 The Interacting String

Let us now consider the interactions of oriented open bosonic strings, and study the dynamics of Chan-Paton degrees of freedom attached to their ends.

Let v_κ be the highest weight vector of the representation R in V, satisfying

$$R(h)\,v_\kappa = \kappa(h)\,v_\kappa \quad \forall h\in\mathcal{H}, \qquad {v_\kappa}^\dagger v_\kappa = 1. \tag{14}$$

On the one hand, it allows us to rewrite the action, in its euclidean form, as [8]

$$S_\Sigma{}^E[\gamma,x,g] = S_\Sigma{}^E[\gamma,x] - i\int_{\partial\Sigma} d\tau\;\kappa\big(-ig^{-1}\partial_\tau g\big) \tag{15}$$

$$= S_\Sigma{}^E[\gamma,x] - i\int_{\partial\Sigma} d\tau\;\big({v_\kappa}^\dagger R(-ig^{-1}\partial_\tau g)v_\kappa\big). \tag{16}$$

On the other hand, it enables us to build from the vertex functional associated with the naked state $|X\rangle$

$$V_\Sigma{}^{(X)}[\gamma,x] = \int_{\partial\Sigma} d\tau\;\mathcal{V}^{(X)}[\gamma,x] \tag{17}$$

a new vertex functional associated with the state $\lambda_{ij}\,|X\rangle$

$$V_\Sigma{}^{(\lambda X)}[\gamma,x,g] = \int_{\partial\Sigma} d\tau\;\big({v_\kappa}^\dagger R^\dagger(g)\lambda R(g)v_\kappa\big)\;\mathcal{V}^{(X)}[\gamma,x]. \tag{18}$$

The new vertex functional remains covariant under Poincaré transformations of the spacetime coordinates, and stays invariant under orientation preserving world-sheet diffeomorphisms and Weyl transformations of the metric. It is also covariant

$$V_\Sigma{}^{(\lambda^U X)}[\gamma, x, g^U] = V_\Sigma{}^{(\lambda X)}[\gamma, x, g] \tag{19}$$

under the left action of G on $g(\tau)$

$$g(\tau) \;\to\; g^U(\tau) = U g(\tau), \qquad \lambda_{ij}\,|X\rangle \;\to\; \lambda_{ij}{}^U\,|X\rangle = (R(U)\lambda R^\dagger(U))_{ij}\,|X\rangle. \tag{20}$$

Now we can define the S-matrix element describing the interactions of S external states $\lambda^s{}_{ij}\,|X^s\rangle$ as the euclidean path integral

$$\mathrm{S}\big(\lambda^1 X^1,\; \ldots,\; \lambda^S X^S\big) = i \sum_\Sigma \int [\mathcal{D}\gamma] \int [\mathcal{D}x] \int [\mathcal{D}g]\; \mathrm{g}^{-\mathcal{X}_\Sigma}\; e^{-S_\Sigma{}^E[\gamma,x,g]} \;\prod_{s=1}^{S} V_\Sigma{}^{(\lambda^s X^s)}[\gamma, x, g]. \tag{21}$$

The product of the vertex functionals $V_\Sigma{}^{(\lambda^s X^s)}[\gamma, x, g]$ associated with the external states is weighted by the exponential of the euclidean action $S_\Sigma{}^E[\gamma, x, g]$ and a power $\mathrm{g}^{-\mathcal{X}_\Sigma}$ of the string coupling, where $\mathcal{X}_\Sigma$ denotes the Euler characteristics of the world-sheet. Then the result is summed over all compact oriented world-sheets Σ, euclidean metrics $\gamma_{\alpha\beta}(\sigma)$, space-time embeddings $x^\mu(\sigma)$ and Chan-Paton degrees of freedom $g(\tau)$, inequivalent under orientation preserving world-sheet diffeomorphisms and Weyl transformations of the metric.

In order to give a meaning to the path integral over the Chan-Paton degrees of freedom

$$\int [\mathcal{D}g]\; e^{i\int_{\partial\Sigma} d\tau\; \left(v_\kappa{}^\dagger R(-ig^{-1}(\tau)\partial_\tau g(\tau))v_\kappa\right)} \;\prod_{s=1}^{S} \big(v_\kappa{}^\dagger R^\dagger(g(\tau^s))\lambda^s R(g(\tau^s))v_\kappa\big), \tag{22}$$

we choose discrete values τ_n of the continuous parameter τ describing the boundary of the string, some of which, τ_{n^s}, coincide with the insertion points τ^s of the vertex functionals. Following [8], we then define the path integral as the continuum limit of the expression

$$\prod_n \Big(N_R \int \mathcal{D}g_n\Big)\; \prod_n \Big(1 + i(\tau_{n+1} - \tau_n)\big(v_\kappa{}^\dagger R(-ig_n{}^{-1}\frac{g_{n+1} - g_n}{\tau_{n+1} - \tau_n})v_\kappa\big)\Big) \tag{23}$$

$$\times \prod_{s=1}^{S} \frac{(v_\kappa{}^\dagger R^\dagger(g_{n^s})\lambda^s R(g_{n^s+1})v_\kappa)}{(v_\kappa{}^\dagger R^\dagger(g_{n^s})R(g_{n^s+1})v_\kappa)}$$

$$= \prod_n \Big(N_R \int \mathcal{D}g_n\Big)\; \prod_n \big(v_\kappa{}^\dagger R^\dagger(g_n)R(g_{n+1})v_\kappa\big)\; \prod_{s=1}^{S} \frac{(v_\kappa{}^\dagger R^\dagger(g_{n^s})\lambda^s R(g_{n^s+1})v_\kappa)}{(v_\kappa{}^\dagger R^\dagger(g_{n^s})R(g_{n^s+1})v_\kappa)}, \tag{24}$$

where $\mathcal{D}g$ is the Haar measure on G. But the operator

$$N_R \int \mathcal{D}g\; R(g)v_\kappa v_\kappa{}^\dagger R^\dagger(g), \tag{25}$$

which commutes with all operators $R(U)$ and has trace N_R, defines a resolution of unity on V. Thus the discretized form of the path integral, which consists in a product of such operators with the Chan-Paton factors inserted at some places, reduces simply to the expression

$$\mathrm{Tr}\;\mathrm{T}_{\partial\Sigma}\;\prod_{s=1}^{S}\lambda^s, \tag{26}$$

where the time ordering $\mathrm{T}_{\partial\Sigma}$ indicates that the cyclic order of the Chan-Paton factors in the trace must follow that of the insertions of vertex functionals along the boundary of the string.

We finally recover the usual insertion of traces of Chan-Paton factors in the amplitudes describing the interactions of oriented open bosonic strings [2, 3].

5 The Unoriented Case

Let us now extend our analysis to the case of an unoriented open bosonic string.

Now the world-sheet Σ of the string is not oriented, and its boundary $\partial\Sigma$ can be given an arbitrary orientation. So any diffeomorphism $\tilde{\sigma}^\alpha(\sigma)$ of Σ, which must leave the action $S[\gamma, x, g]$ invariant, induces a diffeomorphism $\tilde{\tau}(\tau)$ of $\partial\Sigma$ satisfying

$$\tilde{\sigma}^\alpha\big(\bar{\sigma}(\tau)\big) = \bar{\sigma}^\alpha\big(\tilde{\tau}(\tau)\big), \tag{27}$$

which either preserves ($d\tilde{\tau}/d\tau > 0$) or reverses ($d\tilde{\tau}/d\tau < 0$) this orientation. In the second case, the natural transformation of the Chan-Paton degrees of freedom

$$g(\tau) \;\rightarrow\; \tilde{g}(\tilde{\tau}) = g(\tau(\tilde{\tau})) \tag{28}$$

reverses the sign of the boundary term of the action. To compensate for this sign change, we must choose $\kappa(.)$ in such a way that it is conjugated to $-\kappa(.)$ by an element W in G

$$\kappa(W^{-1}.\,W) = -\kappa(.), \tag{29}$$

which can be taken in the Weyl group of $\mathcal{H}$. Then the modified transformation

$$\begin{aligned} g(\tau) \;\rightarrow\; \tilde{g}(\tilde{\tau}) \;&=\; g(\tau(\tilde{\tau})) \qquad \text{if } \frac{d\tilde{\tau}}{d\tau} > 0, \\ &=\; g(\tau(\tilde{\tau}))W \qquad \text{if } \frac{d\tilde{\tau}}{d\tau} < 0, \end{aligned} \tag{30}$$

leaves the boundary term of the action invariant.

In order to quantize the system successfully, we know that $\kappa(.)$ must correspond to the highest weight of a representation R of G, $-\kappa(.)$ being conjugated to the highest weight of the dual representation R^*. But as $\kappa(.)$ is now conjugated to $-\kappa(.)$, the highest weights of the two representations coincide, and the representations R and R^* are unitarily equivalent. This means that there exists a unitary operator M from V^* to V such that

$$MR^*(T_a) = -R(T_a)M, \qquad MR^*(U) = R(U)M. \tag{31}$$

Then M^* is a unitary operator from V to V^* such that

$$M^*R(T_a) = -R^*(T_a)M^*, \qquad M^*R(U) = R^*(U)M^*. \tag{32}$$

The operators M and M^* are unique up to a phase, implying the relations

$$M^T = \alpha M, \qquad M^\dagger = \alpha M^*, \tag{33}$$

with $\alpha = \pm 1$. Moreover it is possible to choose a basis of V such that M reduces to

$$M = I \tag{34}$$

in the first case, or to

$$M = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \tag{35}$$

in the second case, with N_R even. The representation R is then said to be real or pseudoreal, and the operators $R(U)$ in V form a subgroup of $SO(N_R)$ or $USp(N_R)$.

The physical quantum states of a free unoriented open bosonic string must be invariant under world-sheet parity, which takes (σ, τ) to $(\pi - \sigma, \tau)$ and thus exchanges the two ends of the string. The Chan-Paton degrees of freedom transform as

$$\tilde{g}_0(\tau) = g_\pi(\tau) W, \qquad \tilde{g}_\pi(\tau) = g_0(\tau) W, \tag{36}$$

inducing the transformation of the Noether charges

$$\tilde{G}_{0a}(\tau) = G_{\pi a}(\tau), \qquad \tilde{G}_{\pi a}(\tau) = G_{0a}(\tau). \tag{37}$$

At the quantum level, the corresponding unitary operator $\hat{\Omega}$ must satisfy

$$\hat{\Omega} \hat{G}_{0a} = \hat{G}_{\pi a} \hat{\Omega}, \qquad \hat{\Omega} \hat{G}_{\pi a} = \hat{G}_{0a} \hat{\Omega}, \tag{38}$$

while reproducing the usual transformation of the naked states $|X\rangle$

$$\hat{\Omega} |X\rangle = (-1)^{N_X} |X\rangle, \tag{39}$$

with N_X measuring their level of excitation. It is then determined up to a phase ε

$$\begin{aligned} \hat{\Omega} \lambda_{ij} \, |X\rangle &= \varepsilon (-1)^{N_X} M_{ij'} {M^*}_{ji'} \lambda_{i'j'} \, |X\rangle \\ &= \varepsilon (-1)^{N_X} (M \lambda^T M^\dagger)_{ij} \, |X\rangle. \end{aligned} \tag{40}$$

Thus states are invariant under world-sheet parity

$$\hat{\Omega} \lambda_{ij} \, |X\rangle = \lambda_{ij} \, |X\rangle \tag{41}$$

if the associated Chan-Paton factors verify the condition

$$(M \lambda^T M^\dagger)_{ij} = \varepsilon (-1)^{N_X} \lambda_{ij}, \tag{42}$$

where the phase ε have to be restricted to ± 1 to insure their existence.

The vertex functionals associated with the physical quantum states of an unoriented open bosonic string must be invariant under any world-sheet diffeomorphism $\tilde{\sigma}^\alpha(\sigma)$, in particular when the corresponding boundary diffeomorphism $\tilde{\tau}(\tau)$ is orientation reversing ($d\tilde{\tau}/d\tau < 0$). Recalling the transformation law of the naked vertex functional

$$V^{(X)}[\tilde{\gamma}, \tilde{x}] = (-1)^{N_X} V^{(X)}[\gamma, x], \tag{43}$$

we have

$$V_\Sigma{}^{(\lambda X)}[\tilde{\gamma}, \tilde{x}, \tilde{g}] = \int_{\partial\Sigma} d\tilde{\tau} \, \left({v_\kappa}^\dagger R^\dagger(\tilde{g}) \lambda R(\tilde{g}) v_\kappa \right) \, \mathcal{V}^{(X)}[\tilde{\gamma}, \tilde{x}] \tag{44}$$

$$= (-1)^{N_X} \int_{\partial\Sigma} d\tau \, \left({v_\kappa}^\dagger R^\dagger(W) R^\dagger(g) \lambda R(g) R(W) v_\kappa \right) \, \mathcal{V}^{(X)}[\gamma, x] \tag{45}$$

$$= (-1)^{N_X} \int_{\partial\Sigma} d\tau \, \left({v_\kappa}^{*\dagger} R^{*\dagger}(W) R^{*\dagger}(g) \lambda^T R^*(g) R^*(W) {v_\kappa}^* \right) \, \mathcal{V}^{(X)}[\gamma, x]. \tag{46}$$

But the vectors $R^*(W){v_\kappa}^*$ and $M^\dagger v_\kappa$ in V^*, which satisfy

$$- R^*(h)\, R^*(W){v_\kappa}^* = \kappa(h)\, R^*(W){v_\kappa}^* \quad \forall h \in \mathcal{H} \tag{47}$$

and

$$- R^*(h)\, M^\dagger v_\kappa = \kappa(h)\, M^\dagger v_\kappa \quad \forall h \in \mathcal{H}, \tag{48}$$

are both highest weights vectors of the representation R^*, and are therefore equal up to a phase. So we have again

$$V_\Sigma{}^{(\lambda X)}[\tilde{\gamma},\tilde{x},\tilde{g}] = (-1)^{N_X} \int_{\partial\Sigma} d\tau\ \big({v_\kappa}^\dagger M R^{*\dagger}(g)\lambda^T R^*(g) M^\dagger v_\kappa\big)\ \mathcal{V}^{(X)}[\gamma,x] \tag{49}$$

$$= (-1)^{N_X} \int_{\partial\Sigma} d\tau\ \big({v_\kappa}^\dagger R^\dagger(g) M \lambda^T M^\dagger R(g) v_\kappa\big)\ \mathcal{V}^{(X)}[\gamma,x]. \tag{50}$$

Thus vertex functionals are invariant under any world-sheet diffeomorphism

$$V_\Sigma{}^{(\lambda X)}[\tilde{\gamma},\tilde{x},\tilde{g}] = V_\Sigma{}^{(\lambda X)}[\gamma,x,g] \tag{51}$$

if the associated Chan-Paton factors verify the condition

$$(M\lambda^T M^\dagger)_{ij} = (-1)^{N_X}\lambda_{ij}, \tag{52}$$

where the ambiguous sign ε is now fixed to 1.

We finally recover the usual constraint on the Chan-Paton factors associated with the quantum states of an unoriented open bosonic string [2, 3].

6 String in an External Yang-Mills Gauge Field

Let us finally take advantage of the Chan-Paton degrees of freedom attached to the ends of an open bosonic string to couple it to an external Yang-Mills gauge field $A_\mu(x)$ in $\mathcal{G}$. The action reads [5, 9]

$$S_\Sigma[x,\gamma,g;A] = S_\Sigma[x,\gamma] + \int_{\partial\Sigma} d\tau\ \kappa\big(-ig^{-1}(\partial_\tau + iA_\tau)g\big), \tag{53}$$

where

$$A_\tau(\tau) = A_\mu\big(x(\bar{\sigma}(\tau))\big)\,\partial_\alpha x^\mu(\bar{\sigma}(\tau))\,\partial_\tau\bar{\sigma}^\alpha(\tau) \tag{54}$$

is the pullback of the gauge field on the boundary of the string. It is invariant

$$S_\Sigma[x,\gamma,g^U;A^U] = S_\Sigma[x,\gamma,g;A] \tag{55}$$

under the gauge transformations

$$A_\mu{}^U(x) = U(x)A_\mu(x)U^{-1}(x) - iU(x)\partial_\mu U^{-1}(x), \tag{56}$$

$$g^U(\tau) = U\big(x(\bar{\sigma}(\tau))\big)g(\tau), \tag{57}$$

and generalizes the well-known coupling to an abelian gauge field [10, 11].

We can define the effective action for an external Yang-Mills gauge field $A_\mu(x)$ in the background of interacting open bosonic strings as the euclidean path integral

$$\mathrm{S}[A] = i\sum_\Sigma \int[\mathcal{D}\gamma]\int[\mathcal{D}x]\int[\mathcal{D}g]\ \mathrm{g}^{-\mathcal{X}_\Sigma}\ e^{-S_\Sigma{}^E[\gamma,x,g;A]}, \tag{58}$$

in which the cancelation of Weyl anomalies gives rise to equations of motion for the gauge field. Following [8], we define the path integral over the Chan-Paton degrees of freedom

$$\int [\mathcal{D}g] \; e^{i \int_{\partial\Sigma} d\tau \; \left(v_\kappa{}^\dagger R(-ig^{-1}(\tau)(\partial_\tau + A_\tau(\tau))g(\tau)) v_\kappa \right)} \tag{59}$$

as the continuum limit of the expression

$$\prod_n \left(N_R \int \mathcal{D}g_n \right) \; \prod_n \left(1 + i(\tau_{n+1} - \tau_n) \left(v_\kappa{}^\dagger R(-ig_n{}^{-1} \frac{g_{n+1} - g_n}{\tau_{n+1} - \tau_n} + g_n{}^{-1} A_n g_{n+1}) v_\kappa \right) \right) \tag{60}$$

$$= \prod_n \left(N_R \int \mathcal{D}g_n \right) \; \prod_n \left(v_\kappa{}^\dagger R^\dagger(g_n)(1 + i(\tau_{n+1} - \tau_n) R(A_n)) R(g_{n+1}) v_\kappa \right) \tag{61}$$

$$= \mathrm{Tr} \; \prod_n (1 + i(\tau_{n+1} - \tau_n) R(A_n)). \tag{62}$$

It reduces to a discretized form of the trace of the Wilson loop of the gauge field along the boundary of the string

$$\mathrm{Tr} \; \mathrm{T}_{\partial\Sigma} \; e^{i \int_{\partial\Sigma} d\tau \;\; R(A_\tau(\tau))}. \tag{63}$$

We finally recover the usual definition of the effective action for an external Yang-Mills gauge field in the background of interacting open bosonic strings [11, 12].

7 Conclusion

In this paper, we studied a new group-valued degree of freedom attached to the boundary of an open bosonic string. First we showed that taking it into account in the canonical quantization of a free string gives rise to the famous Chan-Paton factors, while introducing it in the path integral description of string interactions leads to the usual insertion of traces of such factors in the amplitudes. Then we explained how this approach reproduces the traditional constraint on these factors in the unoriented case. Finally we indicated how it can be used to describe the coupling of an open bosonic string to an external Yang-Mills gauge field.

Our approach thus provides a natural way to introduce non-abelian interactions in open bosonic string theory, starting from a classical action, and reproducing the Chan-Paton construction at the quantum level. It must naturally be completed by further well-known results, such as the necessity to choose the full symmetry groups $U(N)$, $SO(N)$ or $USp(N)$ in their fundamental representation to insure unitarity at the tree level, or even more severe restrictions imposed by quantum consistency at the one-loop level.

The discussion could be extended in many ways. Attaching Chan-Paton degrees of freedom to the boundary of type I superstrings should give similar results. The study of their T-dual picture could give a new insight on the interaction of strings with D-branes.

Acknowledgements

I thank Professor J. Govaerts for his careful reading of the manuscript. This work was supported by the National Fund for Scientific Research (F.N.R.S., Belgium) through an Aspirant Research Fellowship, and by the Institut Interuniversitaire des Sciences Nucléaires and the Belgian Federal Office for Scientific, Technical and Cultural Affairs through the Interuniversity Attraction Poles (IAP) P6/11.

References

[1] H.M. Chan, J.E. Paton, *Generalized Veneziano Model with Isospin,* Nucl. Phys. B10, 516 (1969).

[2] M.B. Green, J.H. Schwarz, E. Witten, *Superstring Theory,* Volume 1, Cambridge University Press, Cambridge (1987).

[3] J. Polchinski, *String Theory*, Volume 1 : *An Introduction to the Bosonic String,* Cambridge University Press, Cambridge (1998).

[4] N. Marcus, A. Sagnotti, *Group Theory from "Quarks" at the Ends of Strings*, Phys. Lett. B188, 58 (1987).

[5] A.P. Balachandran, S. Borchardt, A. Stern, *Lagrangian and Hamiltonian Descriptions of Yang-Mills Particles*, Phys. Rev. 17, 3247 (1978).

[6] R. Jackiw, V.P. Nair, S.Y. Pi, A.P. Polychronakos, *Perfect Fluid Theory and its Extensions*, J. Phys. A37, 327 (2004).

[7] D.I. Diakonov, V.Y. Petrov, *A Formula for the Wilson Loop,* Phys. Lett. B224, 131 (1989).

[8] B. Broda, *Non-abelian Stokes Theorem in Action,* in M.W. Evans (ed.), *Modern Nonlinear Optics,* Part 2 (2nd edition), Wiley, 429-458 (2001).

[9] B.M. Barbashov, A.P. Koshkarov, *Open String in Background Non-Abelian Field*, Th. Math. Phys. 85, 1134 (1990).

[10] A. Abouelsaood, C.G. Callan, C.R. Nappi, S.A. Yost, *Open Strings in Background Gauge Fields*, Nucl. Phys. B280, 599 (1987).

[11] E.S. Fradkin, A.A. Tseytlin, *Non-linear Electrodynamics from Quantized Strings,* Phys. Lett. B163, 123 (1985).

[12] H. Dorn, H.J. Otto, *Open Bosonic Strings in General Background Fields,* Z. Phys. C32, 599 (1986).